

# When dunes move together, structure of deserts emerges

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Crescent shaped barchan dunes are highly mobile dunes that are usually presented as a prototypical model of sand dunes. Although they have been theoretically shown to be unstable when considered separately, it is well known that they form large assemblies in desert. Collisions of dunes have been proposed as a mechanism to redistribute sand between dunes and prevent the formation of heavily large dunes, resulting in a stabilizing effect in the context of a dense barchan field. Yet, no models are able to explain the spatial structures of dunes observed in deserts. Here, we use an agent-based model with elementary rules of sand redistribution during collisions to access the full dynamics of very large barchan dune fields. Consequently, stationary, out of equilibrium states emerge. Triggering the dune field density by a sand load/lost ratio, we show that large dune fields exhibit two asymptotic regimes: a dilute regime, where sand dune nucleation is needed to maintain a dune field, and a dense regime, where dune collisions allow to stabilize the whole dune field. In this dense regime, spatial structures form: the dune field is structured in narrow corridors of dunes extending in the wind direction, as observed in dense barchan deserts.

In contrast with the layman's view, not all deserts are vast sand seas. Depending on the variability of the local winds, sand dunes can adopt various shapes. When viewed from above, they mimic large stars, long linear ridges or crescent structures [1, 2]. The crescent shaped dune, called barchan [1, 3], is a prototypical model of sand dune dynamics [4] and its properties as an isolated object are now well understood [5, 6]. However, barchans are usually found in large dune assembly, counting tens of thousands of dunes [1, 7, 8]. Barchan fields are observed in regions where a rocky, non-erodible floor is blown by a prevalent unidirectional flow and are ubiquitous on Earth, on Mars and even underseas. The very existence of a barchan field is in apparent contradiction with the fact that barchan dynamic displays an unstable fixed point. Dunes will either grow or shrink if their size departs from their equilibrium size, set by the balance between sand loss and sand capture. In contrast with this unstable behavior, dune size ranges from a few meters to several hundred of meters within a field. The size distribution does not display a lack of small dunes or an anomalous number of huge barchans [7, 9, 10]. Furthermore, barchan fields may spatially be structured in narrow corridors, which extend in the wind direction. These corridors organize the dune field in stripes of dense (resp. diluted) barchan areas, where dunes are smaller (resp. larger), while neither the local conditions such as wind velocity or granulometry, neither the boundary conditions differ [7]. Thus, it is commonly thought that dune-dune interaction, such as dune collisions, are at play to sustain a dune field over longtime, to structure the field in corridors and to select the dune size. Field studies

and underwater experiments have shown that dunes can indeed exchange sands during collision events [11–13]. Dune collision led to merge and split mechanisms, which can be stabilizing providing that large dunes are regularly split into smaller ones, as proposed using a mean field approach [13]. Although this idea is a first step, it is not enough to fully understand how collisions set the structure and affect the stability of a large assembly of dunes. Numerical studies implementing phenomenological rules of collisions have been runned in order to forecast the statistical properties of a large dune field in which collisions take place [9]. If they successfully recover a size selection, the critical aspect of spatial organization of dunes in corridors has never been taken in consideration yet. It is a general problem in which the large scale emergent property comes from the complex, local, interactions of many objects, what perfectly falls within the scope of an agent-based model.

Here we use an agent-based model, implementing dune collisions over the whole dune field, to infer the dynamical statistical behavior of a large dune field in the limit of long observation time, its stationary state and the possible emergence of spatial structures. In order to identify the physical mechanisms involved in the emergent properties of dune fields, this model is restricted to the minimum ingredients of dunes dynamics and interactions. The barchan shape is characterized by a low-slope upwind back and an avalanche face downwind, which is framed by two arms pointing in the wind direction (see Fig. 1(a-b)). The width, length and height of barchans are linearly related to each others such that their morphological state can be defined by one parameter only. Sand

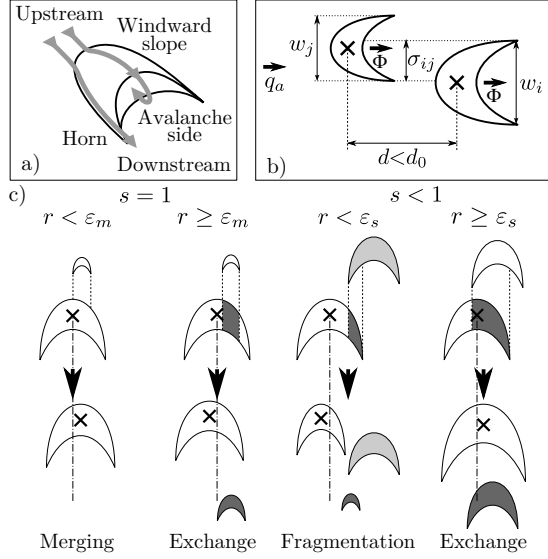


FIG. 1: Elementary rules for barchan collisions (a) Morphology of a barchan. (b) Parameters used to describe a collision: two dunes of size  $w_i$  and  $w_j$  interact when they are closer than  $d_0$  and within a cross section of  $\sigma_{ij}$ . Each dune is losing a volume per unit of time  $\Phi$  and it is fed by sand influx  $q_a$ . (c) The four cases of binary dune collisions. Depending on the upward projections  $s = \frac{\sigma_{ij}}{w_j}$ , the collision is total ( $s = 1$ ) or lateral ( $s < 1$ ). Then, the downward projections  $r = \frac{\sigma_{ij}}{w_i}$  regarding to a merging  $\varepsilon_m$  or a splitting  $\varepsilon_s$  threshold is taken into account. The volume of sand is conserved during a collision. Grey levels show the sand redistribution.

erosion and deposition processes force barchans to move downwind. Their propagation velocity, which increases with the wind shear stress and decreases with the sediment influx, is inversely proportional to the dune size. The avalanche face acts as a sand trap and barchans can propagate over long distances without losing much sand. Yet, small sand loss occurs at the tip of the barchan arms. Starting with a non-null value, the sand loss increases very weakly with increasing dune size, and can be considered as a constant [14]. On the other hand, the input sand flux is proportional to the dune width. As a result, the fixed point (where loss and gain are balanced) is unstable and an isolated barchan can only grow or shrink and eventually, disappear [14]. Indeed, below a critical size, the barchan loses its avalanche face, turns into a dome-like structure and quickly vanishes.

In our model, dunes are described by their width  $w$  only and, for the sake of simplicity, are cubic. They propagate downwind at a speed  $v$ :

$$v = \frac{\alpha}{w}. \quad (1)$$

We assume that dunes lose sand homogeneously along their downwind face. We call  $\Phi$  the volume lost per unit of time because of wind erosion. Barchans can also grow,

due to an incoming sand influx per unit of length transverse to the wind,  $q_a$ . The volume  $V$  of an isolated dune will then vary in function of time as:

$$\frac{dV}{dt} = -\Phi + q_a w. \quad (2)$$

Equation 2 contains the fundamental instability of one isolated barchan of unstable equilibrium size  $\tilde{w} = \Phi/q_a$ . If the dune shrinks below the dome size  $w_c$ , it is removed from the field. The model does not conserve the mass.

One hypothesis to reconcile the unstable behavior of an isolated dune (eq. 2) with the existence of dense barchan field is to consider dune collisions. Smaller dunes are faster (eq. 1) and can collide with larger, slower dunes what leads to a transfer of mass between dunes [9, 13]. Dune collisions led to merge and split mechanism depending on the relative size of dunes and their lateral alignment. Numerical studies have shown that those parameters set the result of a collision [15–17]. If the incoming dune is very small, it is simply absorbed by the larger, slower one. If the dunes are of similar sizes, a redistribution of mass occurs, and one (or several) small dunes are emitted at the front while a larger dune is formed at the back.

In our model, two dunes are in interaction if they are closer than a distance  $d_0$  in the wind direction and if their width projections overlap as shown in figure 1b. We consider two types of interactions: a distant one through emitted sand capture and sand flux screening and a close one through collision. The distance  $d_0$  reflects a typical distance for the sand flux to get diluted laterally. Let's consider two dunes  $i$  and  $j$ ,  $i$  being the downwind dune and  $j$  the upwind one. The size of  $i$  is noted  $w_i$  and the overlapped width  $\sigma_{ij}$ . We define the upward projection  $s = \frac{\sigma_{ij}}{w_j}$  and the downward projection  $r = \frac{\sigma_{ij}}{w_i}$ . The leeward dune catches a part  $s$  of the sand lost by the upwind dune. In the same time, the upwind dune screens the leeward dune on the width  $\sigma_{ij}$  from any flux coming upwind of  $i$  and  $j$ . If the upwind dune  $j$  is the only one that is closer than  $d_0$  to the downwind dune  $i$ , the volume of the latter varies as  $dV_i/dt = s \times \Phi + q_a(w_i - \sigma_{ij}) - \Phi$ . Note that these eolian mass exchanges do not affect the aspect ratio of the dune (cubic) nor their position.

Two dunes collide when they overlap following the rules shown in Fig. 1c. When  $s=1$  (perfect overlap),  $r$  is the size ratio. The two dunes merge if  $r$  is smaller than the merging threshold  $\varepsilon_m$ , and the new dune gets a volume:  $V_i^{t+\Delta t} = (w_i^t)^3 + (w_j^t)^3$ . When  $s=1$  and  $r \geq \varepsilon_m$ , the total sand is redistributed into two new dunes of volume:  $V_i^{t+\Delta t} = (w_i^t)^3 + (w_j^t)^3 - \sigma_{ij}w_i^2$  and  $V_j^{t+\Delta t} = \sigma_{ij}w_i^2$ . When  $s < 1$  (partial overlap), the sand is redistributed into two or three dunes, respectively for  $r$  values bigger or smaller than the splitting threshold  $\varepsilon_s$ . When  $s < 1$  and  $r \geq \varepsilon_s$ , the two dunes exchange sand the same way as when  $s=1$  and  $r \geq \varepsilon_m$ . When  $s < 1$  and  $r < \varepsilon_s$ , the bumping dune  $i$  is unaffected while the

$d_0$	$w_0$	$w_c$	$\varepsilon_s$	$\varepsilon_m$	$\lambda^{-1}$	$\alpha$	$\Phi \times 10^7$	$q_a$	$\ell$	$L$	$\Delta t$
1	0.1	0.01	0.5	0.5	2048	$10^{-3}$	[1.5;500]	0	32	[32;128]	1

TABLE I: Parameters of the simulations.

bumped dune is split in two dunes. The ejected dune  $k$  gets a volume:  $V_k^{t+\Delta t} = \sigma_{ij} w_i^2$  while  $V_j^{t+\Delta t} = (w_j^t)^3$  and  $V_i^{t+\Delta t} = (w_i^t)^3 - \sigma_{ij} w_i^2$ . Note that the centers of mass of the new dunes are set at the barycentric positions of the incoming sand, which may shift the dunes laterally. To compensate for sand dune loss, dunes can appear by nucleation anywhere in the dune field where there is an empty place with a probability per unit time and per unit of surface  $\lambda$ . Their size is arbitrary set to  $w_0$ . These nucleations are the trace of topographical defects that promotes sand deposition [1, 2]. This choice maximizes the effect of noise. Thus any emerging behaviors will be robust.

Looking for stationary behavior of large dune field, the dune field boundaries are periodic and the field is long compared to the typical distance of dune interactions,  $L \gg d_0$ . Since, dunes move along the wind direction, we expect that the perpendicular direction  $l$ , does not play a major role in dune-dune dynamics. The field is initially filled with dunes at random positions homogeneously chosen. Their sizes follow a constant probability which is centered on  $w_0$  and with a minimum cut-off value of  $w_c$ . The numerical method used to compute the assembly of dunes in this large field is based on synchronous algorithm and off-lattice dynamics as in self-propelled particles models [18]. By their non-trivial kinematics, dunes can indeed be considered as self-propelled particles, which exchange mass – or momentum – with their neighborhood. Values of the different parameters used in the simulations are reported in table I. In particular, we assume that there is little sand around the dunes, so that the ambient influx  $q_a$  is null. It implies that a lonely dune always vanishes. From a simulation to an other, the dune density of the field is triggered through a fixed nucleation rate  $\lambda$  and a changing erosion rate  $\Phi$ .

From the microscopic parameters, seven independent dimensionless numbers can be built. One can define three length ratios ( $w_0/d_0$ ,  $w_c/d_0$ ,  $\tilde{w}/d_0 = \Phi/(q_a d_0)$ —infinite here),  $\varepsilon_s$  and  $\varepsilon_m$  which control the dynamics of collisions and two times ratio. The three relevant times are expected to be the time of disappearance of an isolated dune of size  $w_0$ :  $t_{\text{eol}} = (w_0^3 - w_c^3)/\Phi$ , the typical nucleation time:  $t_{\text{nuc}} = (\lambda \times d_0^2)^{-1}$  and a collision time:  $t_{\text{col}}$ , which could be evaluated as the smallest time for two interacting dunes to collide  $t_{\text{col}} = d_0/(\alpha/w_c - \alpha/w_0)$ . In the present study, all parameters but  $\Phi$  are kept constant (see table I):  $t_{\text{nuc}} = 2048$  and  $t_{\text{col}} \simeq 11$  while  $t_{\text{eol}} \in [20; 6.7 \times 10^3]$ , an isolated dune of size  $w_0$  travels a distance 2 to 660 times its initial size before disappearing.

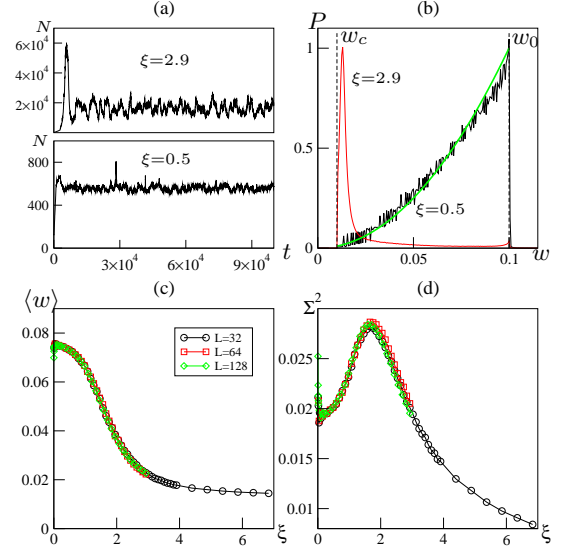


FIG. 2: Characteristics of diluted and dense dune field. (a) number of dunes along time for dense and dilute regime. (b) size distributions for dilute and dense regime. The continuous green line is the analytic law of diluted distribution, eq. 4. (c) mean width  $\langle w \rangle$  and (d) variance of the width  $\Sigma^2$  versus  $\xi$  for different system lengths. Color online.

We define the control parameter  $\xi$ :

$$\xi = \frac{t_{\text{eol}}}{t_{\text{nuc}}} = \frac{w_0^3 - w_c^3}{\Phi} \lambda d_0^2. \quad (3)$$

It measures the balance between disappearance of dunes due to loss of sand and dune nucleation. So, one can expect that a field gets emptied when  $\xi \ll 1$ .

Interestingly, the dune field always reaches a stationary state within our range of parameters (see Fig. 2a). For small  $\xi$ , the barchan field is diluted, a few dunes are dispersed across the whole field and dune collisions are rare. Dunes can be considered as separate, unstable objects whose disappearance is balanced by nucleation only.  $\xi$  is actually the exact dimensionless stationary density in the limit of  $\xi \ll 1$ . The normalized distribution of size  $P(w)$  (see Fig. 2b) follows the analytic distribution:

$$P(w) = \frac{3w^2}{w_0^3 - w_c^3}, \text{ for } w \in [w_c; w_0], \quad (4)$$

which can be derived from the individual dynamics (Eq. 2). Therefore, the typical size of dune is about  $3/4 w_0$ , when  $w_c$  is small enough. Such a field with low interaction between dunes compares to diluted deserts such as the barchan field of La Pampa de la Joya in Peru [7].

At large density  $\xi \geq 1$  collisions dispatch sand in a non-trivial manner. The field state remains stationary (Fig. 2a), but the size distribution is fundamentally altered, and shifted to small sizes (Fig. 2b). Dune collisions tend to increase the number of dunes more rapidly than the effect of nucleation itself. With  $\varepsilon_s$  and  $\varepsilon_m$  set to

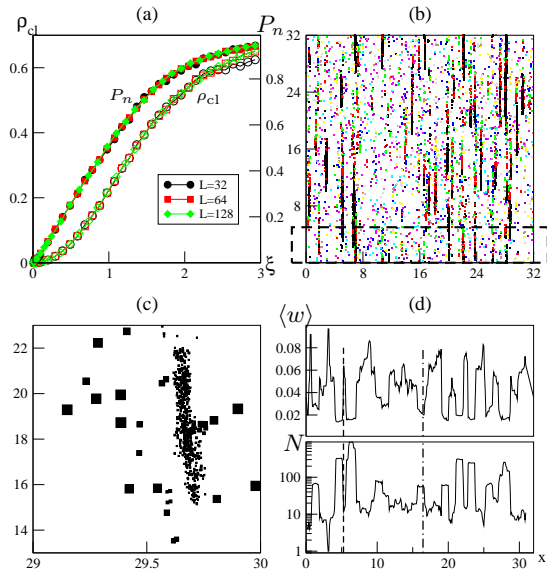


FIG. 3: Spatial organization of a dune field (a) Density of clusters  $\rho_{cl}$  and probability  $P_n$  for a dune to be in a cluster *vs*  $\xi$  for different system sizes. (b) snapshot of a computed desert at  $\xi = 2.9$ , colors stand for dunes size. (c) closeup of a cluster of sub-figure (b). (d) Profiles of mean size  $\langle w \rangle$  and number  $N$  of dunes show anticorrelation, see *e.g.* dashed and dotted-dashed lines. Profiles are computed by averaging in the bold dashed box shown on figure (b). (Color online. Movies are available as supplementary materials)

0.5, the effective predominant collision type is the fragmenting one that creates an additional dune (Fig. 1c). Note that, the most probable size is close but larger than the minimal size of dunes  $w_c$  (Fig. 2b). A dense assembly of dunes is not a trivial homogeneous field with frequent collisions. On the contrary, dense spatial clusters of small interacting dunes (*i.e.* inter-dunes is smaller than  $d_0$ ) develop and gather the major part of the dunes as seen on Fig. 3. These clusters, with sharp boundaries, are elongated in the wind direction, so that the field is self-structured in a corridor-like pattern where the local density is a highly fluctuating quantity. If we restrict our measure to local low density, the dune size distribution shows a maximum at  $w_0$  as in a diluted desert: the local dune size is directly correlated to the local density of dunes. A dense field looks like a dilute field of big dunes with dense corridors of small dunes (see Fig. 3d). These spatial structures and relation between dune size and dilution are similar to what is observed in the long barchan field that extends in the Atlantic Sahara (Morocco, see [7] and supplementary material). Since many small and fast dunes are created, another observed effect of collisions is the spreading of the speed distribution. They can impact larger dunes and lead to a succession of avalanche-like collisions. Thus, transitory times are very different between diluted and dense deserts. Whereas the number of dunes relaxes normally in diluted deserts, dense deserts exhibit

nearly periodic blow-up of their population.

We identified two stationary states (at  $\xi \ll 1$  and  $\xi \geq 1$ ) where the number of dunes but also the size distribution, the spatial arrangement in the field and the relaxation to the equilibrium are dissimilar (fig. 2 and 3). This is the signature of two different field dynamics: a diluted field where dunes barely interact, and a dense field whose dynamics is controlled by dune collisions. It is therefore legitimate to ask whether there is any phase transition when transiting from one to another. We checked that the mean dune size and its variance change continuously when  $\xi$  was varied (Fig 2c and d). More generally, whatever the order parameter we looked at, we found it to change smoothly without any diverging moment. Furthermore, we did not detect any finite size effect (see Fig. 2c and 3a), which could sign a continuous [19] or a first order phase transition [20]. We neither found any influence of initial conditions to the final state nor detect any meta-stability. Even if the two stationary states are very different, there is no phase transition but rather a smooth cross-over when varying  $\xi$  in this set of other fixed parameters. Note that the system we studied is far from equilibrium. Thus a phase transition was allowed even at low dimension, in contrast with systems at equilibrium [21].

In conclusion, we introduced a minimal agent-based model of barchans in interactions, in which kinematics and interactions are set in considering experimental evidence of dune collisions. Its domain of validity cannot extend outside the framework of asymptotic limits: infinite size and infinite time of observations. However, varying the life time of barchans due to sand loss, we showed a smooth cross-over between a diluted desert to a dense desert where dunes aggregate in elongated clusters. Computed deserts self-organize in corridor-like patterns with dense regions of small dunes, and diluted spaces of larger dunes. This is observed in Earth dense barchan fields. Our model, although minimal, was able to capture the emergence of such heterogeneous patterning. In clusters, the typical barchan size is the result of avalanche of collisions. Therefore, we demonstrated that a dune fragmentation mechanism, which seems to lack in previous studies, is a key process in setting the emergent properties of barchan dune fields. This fragmentation mechanism is here provided by collisions, but one could expect that other barchan destabilization mechanisms could play a similar role.

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